

Fourier Transform

* Represent (transform) a non-Periodic Function From time domain to Frequency domain

T.D $\xrightarrow{\text{F.T}}$ F.D

$g(t) \xrightarrow{\text{F.T}} G(F)$

$$G(F) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi Ft} \cdot dt$$

I.F.T.

F.D $\xrightarrow{\text{I.F.T.}}$ T.D

$G(F) \xrightarrow{\text{I.F.T.}} g(t)$

$$g(t) = \int_{-\infty}^{\infty} G(F) \cdot e^{+j2\pi Ft} \cdot dF$$

(1) Find F.T of the rectangular pulse shown.

$$\begin{aligned} g(t) &= A \text{rect}\left(\frac{t}{T}\right) \\ &= A \text{rect}\left(\frac{t}{T}\right) \end{aligned}$$

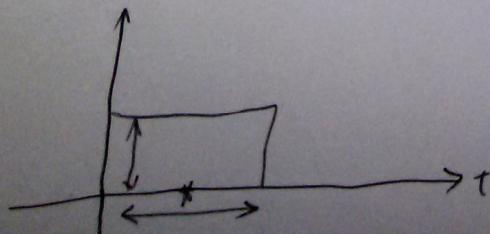
to draw rect

1 - amplitude (A)

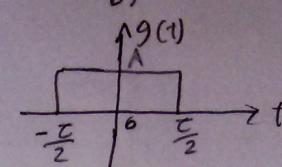
2 - Center

width $\frac{T}{2}$, gives $\text{bw} = \frac{1}{2}$ is $\frac{1}{2}$ in width $\frac{T}{2}$

3 - Width $\frac{T}{2}$



$$A \text{rect}\left(\frac{t}{T}\right)$$



$$G(F) = \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi Ft} \cdot dt$$

$$= \int_{-\frac{T}{2}}^{\frac{T}{2}} A \cdot e^{-j2\pi Ft} \cdot dt$$

$$= \frac{A}{-j2\pi F} \left[e^{-j2\pi F \frac{T}{2}} - e^{+j2\pi F \frac{T}{2}} \right]$$

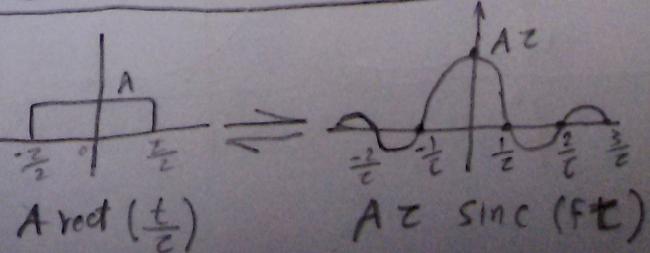
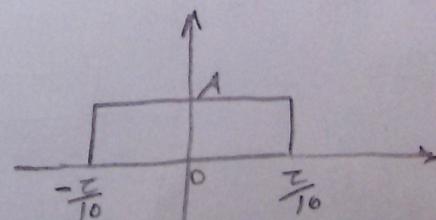
$$= \frac{A}{j\pi F} \left[\frac{e^{+j\pi F \frac{T}{2}} - e^{-j\pi F \frac{T}{2}}}{2j} \right]$$

$$= \frac{A \frac{T}{2}}{j\pi F \frac{T}{2}} \sin(\pi F \frac{T}{2})$$

$$= AT \cdot \text{sinc}(FT)$$

$$A \text{rect}\left(\frac{t}{T}\right) \iff AT \text{sinc}[FT]$$

$$A \text{rect}\left(\frac{3t}{T}\right) = A \text{rect}\left[\frac{t}{T/3}\right]$$



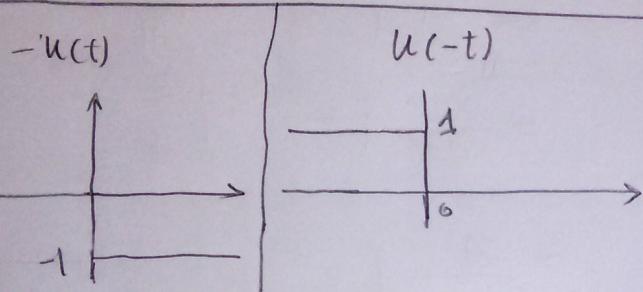
* Some Important Functions:-

① Unit Step Function $U(t)$

$$U(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$u(t) = \begin{cases} 1 & t > 0 \\ \frac{1}{2} & t = 0 \\ 0 & t < 0 \end{cases}$$

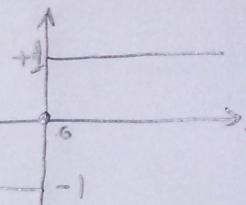
(0, 1) *will cross 0 V just below 0*



② Signum Function $\text{sgn}(t)$

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$

$\text{sgn}(t)$



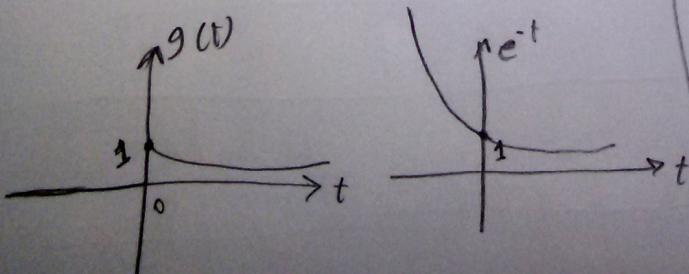
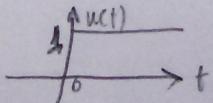
③ Delta Function $\delta(t)$

$$\delta(t) = \begin{cases} \infty & t=0 \\ 0 & t \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

Find f.T. For $g(t) = U(t) \cdot e^{-t}$

* Draw

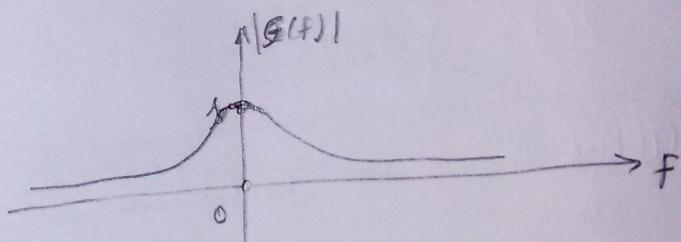


$$\begin{aligned} G(f) &= \int_{-\infty}^{\infty} g(t) \cdot e^{-j2\pi ft} dt \\ &= \int_0^{\infty} e^{-t} \cdot e^{-j2\pi ft} dt \\ &= \frac{1}{-j2\pi f - 1} \left[e^{-t(-j2\pi f + 1)} \right]_0^{\infty} \\ &= \frac{1}{(1 + j2\pi f)} \left[e^{-\infty} - e^0 \right] \\ &= \frac{1}{1 + j2\pi f} \end{aligned}$$

$$U(t) \cdot e^{-t} \iff \frac{1}{1 + j2\pi f}$$

* Draw in F.D

$$|G(f)| = \frac{1}{\sqrt{1 + 4\pi^2 f^2}}$$



2

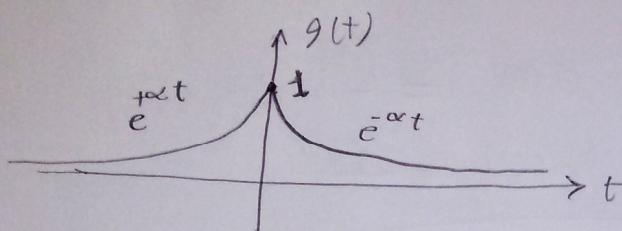
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*Ex: Find F.T. of $g(t) = e^{-\alpha |t|}$

$$|t| = \begin{cases} t & \geq 0 \\ -t & < 0 \end{cases}$$

$$g(t) = \begin{cases} e^{-\alpha t} & t \geq 0 \\ e^{\alpha t} & t < 0 \end{cases}$$



$$\begin{aligned}
 G(f) &= \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt \\
 &= \int_{-\infty}^0 e^{\alpha t} e^{-j2\pi ft} dt + \int_0^{\infty} e^{-\alpha t} e^{-j2\pi ft} dt \\
 &= \frac{e^{t(\alpha - j2\pi f)}}{\alpha - j2\pi f} \Big|_{-\infty}^0 + \frac{e^{-t(\alpha + j2\pi f)}}{\alpha + j2\pi f} \Big|_0^{\infty} \\
 &= \frac{e^0 - e^{\infty}}{\alpha - j2\pi f} - \frac{e^{\infty} - e^0}{\alpha + j2\pi f} \\
 &= \frac{1}{\alpha - j2\pi f} + \frac{1}{\alpha + j2\pi f} \\
 &= \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}
 \end{aligned}$$

$$e^{-\alpha t H} \iff \frac{2\alpha}{\alpha^2 + 4\pi^2 f^2}$$

* Properties of Fourier Transform

فـمـجـمـعـهـمـ النـوـاـصـ تـسـاعـنـاـ فـيـ اـيجـادـ
لـوـالـمـجـهـونـ بـمـعـلـمـاتـ دـوـالـ آـخـرـ وـلـيـسـ عـمـلـهـ

1

Linearity (Superposition)

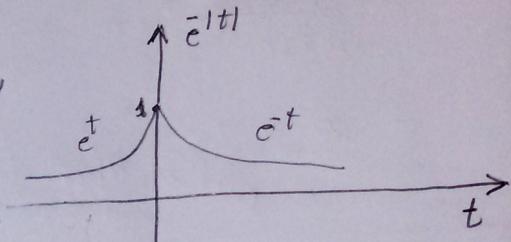
Let $g_1(t) \rightarrow G_1(F)$

$$g_2(t) \rightleftharpoons G_2(f)$$

$$a G_1(t) \pm b G_2(t) \iff a G_1(F) \pm b G_2(F)$$

Ex: Find F.T. for $g(t) = e^{-|t|}$

Using Linearity



$$g(t) = e^{+t} u(-t) + e^{-t} u(t)$$

$$e^{-t} \cdot u(t) \Leftrightarrow \frac{1}{1 + j2\pi f}$$

$$e^t \cdot u(-t) \xrightarrow{\text{?}} \frac{1}{1 - j2\pi f}$$

$$\therefore G(f) = \frac{1}{1+j2\pi f} + \frac{1}{1-j2\pi f}$$

2 Time scaling

$$g(t) \xrightarrow{\quad} G(F)$$

$$g(at) = \frac{1}{|a|} G\left(\frac{x}{a}\right)$$

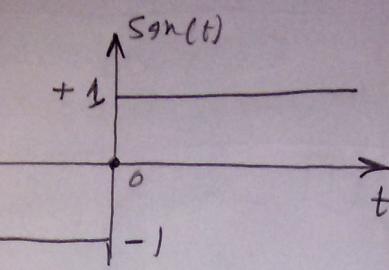
$$E K_i \quad g(t) = e^{-at} \cdot u(t) \quad u(at) = u(t)$$

$$\therefore e^{-t} \cdot u(t) \underset{\sim}{\longrightarrow} \frac{1}{1 + \sqrt{2}\pi f}$$

Using Time Scaling

$$\therefore g(t) = e^{-at} u(at) \Leftrightarrow \frac{1}{|a|} \cdot \frac{1}{1 + j\frac{z\pi}{a}}$$

$$\text{Ex: } g(t) = \text{sgn}(t)$$



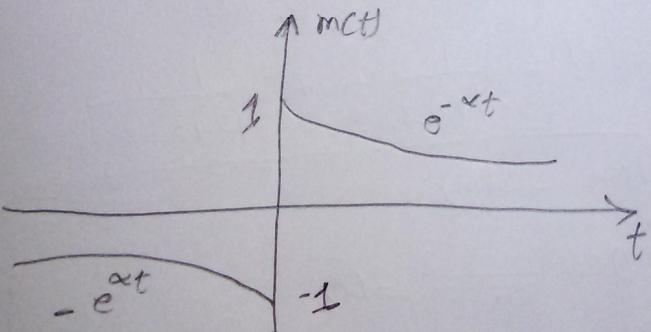
$$\begin{aligned} G(f) &= \int_{-\infty}^0 -1 \cdot e^{-j2\pi ft} dt + \int_0^{\infty} 1 \cdot e^{-j2\pi ft} dt \\ &= -\frac{e^{-j2\pi ft}}{-j2\pi f} \Big|_0^{\infty} + \frac{e^{-j2\pi ft}}{-j2\pi f} \Big|_{-\infty}^0 \\ &= \frac{[e^0 - e^{\infty}]}{j2\pi f} + \frac{e^{\infty} - e^0}{-j2\pi f} \\ &= \infty \end{aligned}$$

~~for which we can do this~~

$m(t)$

$$\text{sgn}(t) = \lim_{\alpha \rightarrow 0} [m(t)]$$

$$\text{F.T.} [\text{sgn}(t)] = \lim_{\alpha \rightarrow 0} [M(\alpha)]$$



$$\text{sgn}(t) = \lim_{\alpha \rightarrow 0} m(t)$$

$$m(t) = -e^{at} \cdot u(-t) + e^{-at} u(t)$$

$$\therefore e^{-at} \cdot u(t) \iff \frac{1}{\alpha + j2\pi f}$$

$$\therefore -e^{at} \cdot u(-t) \iff -\frac{1}{\alpha - j2\pi f}$$

$$\begin{aligned} \text{using super position} \\ M(f) &= \frac{1}{\alpha + j2\pi f} - \frac{1}{\alpha - j2\pi f} \\ &= \frac{-j4\pi f}{\alpha^2 + 4\pi^2 f^2} \end{aligned}$$

$$\text{F.T.} [\text{sgn}(t)] = \lim_{\alpha \rightarrow 0} \frac{-j4\pi f}{\alpha^2 + 4\pi^2 f^2}$$

$$G(f) = -j \frac{1}{\pi f}$$

$$|G(f)| = \frac{1}{\pi f}$$

$$\text{sgn}(t) \iff \frac{1}{j\pi f}$$

$$\text{Ex: } g(t) = u(t)$$

$$\text{sgn} \iff \frac{1}{j\pi f}$$

$$u(t) = \frac{1}{2}[\text{sgn}(t) + 1]$$

using super position

$$u(t) \iff \frac{1}{2} \left[\frac{1}{j\pi f} + \frac{1}{2} \delta(f) \right]$$

$$a \iff a \delta(f)$$

